Application of LQR Techniques to the Adaptive Control of Quasilinear Parabolic PDEs

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joint work with
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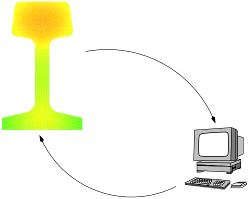
Outline

$\Longrightarrow$



Optimal Cooling of Steel Profiles - A Model Problem

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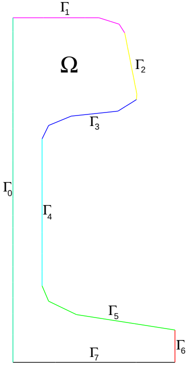
Physical Model

- Physical Model: Cooling of steel profiles in a rolling mill.
  $\Upsilon := \Omega \times (0, T)$ and $\Sigma_i := \Gamma_i \times (0, T)$.

$$\begin{aligned} c(x)\rho(x)\tfrac{\partial}{\partial t}x(\xi,t) &= \nabla.(\lambda(x)\nabla x(\xi,t)) && \text{in } \Upsilon, \\ -\lambda(x)\partial_\nu x(\xi,t) &= g_i(x,u,\xi,t) && \text{on } \Sigma_i, \\ x(\xi,0) &= x_0(\xi) && \text{in } \Omega, \end{aligned}$$

(heat eq.)



- state x temperature
- $c(x)$ specific heat capacity, $\rho(x)$ density, $\lambda(x)$ heat conductivity
- $T \in \mathbb{R} \cup \{\infty\}$ final time. We assume $T = \infty$ here for simplicity.

Source: Physical model: courtesy of Mannesmann/Demag.

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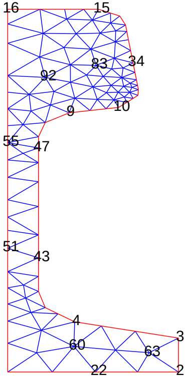
Mathematical Model

- Mathematical model: boundary control for linearized 2D heat equation.

$$\begin{aligned} c \cdot \rho \frac{\partial}{\partial t} x &= \lambda \Delta x, && \xi \in \Omega \\ \lambda \frac{\partial}{\partial n} x &= \kappa(u_k - x), && \xi \in \Gamma_k,\ 1 \leq k \leq 7, \\ \frac{\partial}{\partial n} x &= 0, && \xi \in \Gamma_7. \\ & && \text{(lin. heat eq.)} \end{aligned}$$



- FEM discretization, different models for initial mesh ($n = 371$),
  1, 2, 3, 4 steps of global mesh refinement
  $\Rightarrow n = 1357, 5177, 20209, 79841$.

Source: Math. model: [Tröltzsch/Unger 1999/2001, Penzl 1999, S. 2003]

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Mathematical Model

Control problem

- control heat distribution in Ω
- other examples: heating/cooling processes, air conditioning

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Main interests here:

- use state feedback control, i.e.:

$$\mathbf{u}(t) := \mathbf{F}(\xi, t)\mathbf{x}(\xi, t)$$

- extend the closed loop Riccati approach to nonlinear systems

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LQR Control for the Linear System

abstract Cauchy problem

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \quad \mathbf{x}(0) = \mathbf{x}_0 \in \mathcal{X}. \tag{Cauchy}$$

output equation

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \tag{output}$$

cost function

$$\mathcal{J}(\mathbf{u}) = \frac{1}{2}\int_0^\infty < \hat{\mathbf{Q}}\mathbf{y}, \mathbf{y} > + < \mathbf{R}\mathbf{u}, \mathbf{u} > dt \tag{cost}$$

and the linear quadratic regulator problem is

LQR problem

Minimize the **quadratic** (cost) with respect to the **linear** constraints (Cauchy),(output).

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LQR Control for the Linear System (Lösung)

Well understood in the open literature:
Analogously to the finite dimensional case for $T = \infty$ we find the

optimal feedback control

$$\mathbf{u} = -\mathbf{R}^{-1}\mathbf{B}^*\mathbf{X}_\infty \mathbf{x}.$$

where $\mathbf{X}_\infty$ is the minimal, positive semidefinite, selfadjoint solution of the

algebraic operator Riccati equation

$$0 = \mathcal{R}(\mathbf{X}) := \mathbf{Q} + \mathbf{A}^*\mathbf{X} + \mathbf{X}\mathbf{A} - \mathbf{X}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^*\mathbf{X}. \qquad \text{(ARE)}$$

e.g. [Lions '71; Lasiecka/Triggiani '00; Bensoussan et al. '92; Pritchard/Salamon '87; Curtain/Zwart '95]

Optimal Cooling of Steel Profiles - A Model Problem

LQR Control for the Linear System (Lösung)

Well understood in the open literature:
Analogously to the finite dimensional case for $T<\infty$ we find the

optimal feedback control

$$\mathbf{u} = -\mathbf{R}^{-1}\mathbf{B}^*\mathbf{X}_\infty(t)\mathbf{x}.$$

where $\mathbf{X}_\infty$ is the minimal, positive semidefinite, selfadjoint solution of the

differential operator Riccati equation

$$\dot{\mathbf{X}} = -\mathcal{R}(\mathbf{X}) := -\mathbf{Q} - \mathbf{A}^*\mathbf{X} - \mathbf{X}\mathbf{A} + \mathbf{X}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^*\mathbf{X}. \qquad \text{(DRE)}$$

See talk by H. Mena on the solution of the DREs for details

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LQR Adaptive Control for the Nonlinear Model

Question:

How can we extend this approach to nonlinear systems?

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Idea

1. Linearize the system

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1. Linearize the system
2. Apply the theory above to compute the control

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3. Update the nonlinearity/-ies as the state changes.

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4. Restart at 1

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Here: Linearization $\rightarrow$ Freeze the coefficients/material parameters on certain time intervals

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[Tröltzsch/Unger 1999/2001] applied this idea successfully in the open loop case.

Interpretation of the Model Problem as MPC

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Properties and Ingredients of General MPC Schemes

Properties:

- Model Predictive Control (MPC): class of methods rather than a control technique
- great acceptance in industrial applications
- yields high performance control systems capable of running without expert intervention
- applicable to nonlinear systems

Ingredients

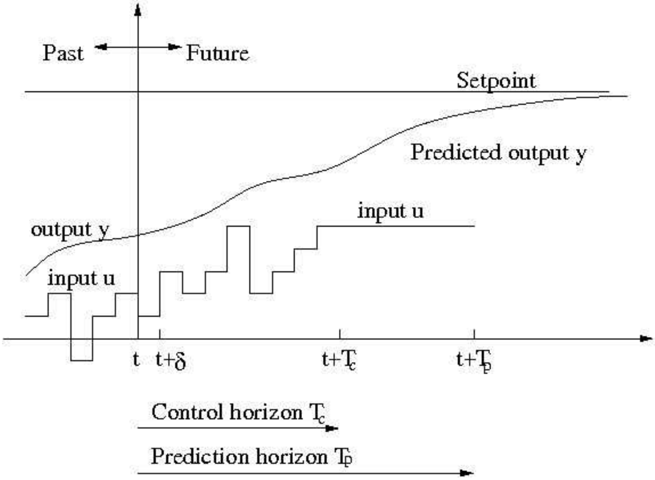
MPC consists of 3 major parts:

1. prediction model
2. cost function
3. way to compute the control

e.g. [Garcia/Prett/Morari '89; Camacho/Bordons '04; Chen/Allgöwer '97/'98]

Interpretation of the Model Problem as MPC

Basic Idea of MPC



Interpretation of the Model Problem as MPC

Identification of Main Ingredients

Following the idea of [Garcia/Prett/Morari '89] nonlinear MPC $\longrightarrow$ linearized optimal control.

Prediction Model

Cost Function

Way to compute the control

($\rightarrow$ example: [Garcia '84] batch reactor application produced excellent results)

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obviously given by the (cost)

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Riccati operator based feedback control for the linearized model

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Prediction Model

nonlinear heat equation (heat eq.)

Cost Function

obviously given by the (cost)

Way to compute the control

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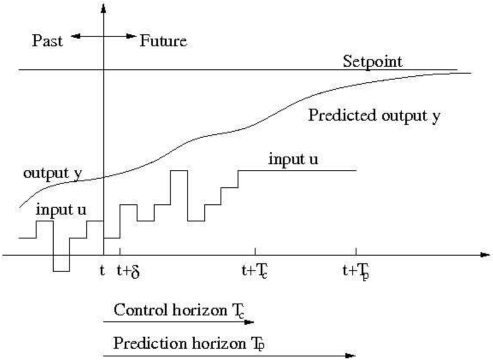
($\longrightarrow$ example: [GARCIA '84] batch reactor application produced excellent results)

Interpretation of the Model Problem as MPC

Identification of Time Intervals



$$
\begin{aligned}
T_C &= \infty. \\
T_P = \delta &= \text{time stepsize on the discrete numerical level}
\end{aligned}
$$

Future Research

- Compare with the DRE case ($T < \infty$) which gives more flexibility in the MPC horizon choices (i.e. $T_C \leq T - t$ and $T_P = \delta$ possibly equal to T_C.) (with H.Mena (EPN Quito))
- Stabilization proof for Steel cooling from MPC proofs
- Sub-optimality error bounds
- Step size control for time discretization schemes
- Interpretation as instantaneous control?

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Thank you for your attention!