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Masterarbeit

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Computational Methods in
Systems and Control Theory

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Computational Methods in Systems and Control Theory

Topic:
Compute the subgradient of the spectral abscissa numerically

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Description:
Many interesting real functions on Euclidean space are differentiable almost everywhere. All Lipschitz functions have this property, but so, for example, does the spectral abscissa of a matrix (a non- Lipschitz function). In practice, the gradient is often easy to compute. This master thesis should investigate to what extent we can approximate the subgradient of the spectral abscissa at some point by calculating the convex hull of some maybe generalized gradients sampled at random nearby points. It should be also part of this thesis to investigate if there is a way to describe the subgradient at points where we do not know how to describe it. In [BLO01] Burke, Lewis and Overton investigate the question of numerically approximating the Clarke subdifferential.

Subgradients: For functions that are not differentiable, subgradients are used instead of the derivative to analyze variational properties. The following definition may be found in [RW98]. Let $\phi : \mathbf{E} \to [-\infty, \infty]$, where $\mathbf{E}$ is a finite-dimensional Euclidean space, real or complex, with the real inner product $\langle \cdot, \cdot \rangle$, and let $x \in \mathbf{E}$ be such that $\phi(x) < \infty$. A vector $y \in \mathbf{E}$ is a **regular subgradient** of ϕ at x (written $y \in \hat{\partial}\phi(x)$) if

$$\liminf_{z \to 0} \frac{\phi(x+z) - \phi(x) - \langle y, z \rangle}{\|z\|} \geq 0. \tag{1}$$

A vector $y \in \mathbf{E}$ is a **subgradient** of ϕ at x (written $y \in \partial\phi(x)$) if there exists sequences x_i and y_i in $\mathbf{E}$ satisfying $x_i \to x$, $\phi(x_i) \to \phi(x)$, $y_i \in \hat{\partial}\phi(x_i)$ and $y_i \to y$.

Examples For the function $\phi(x) = -|x|$ the regular subgradient is given by $\hat{\partial}\phi(0) = \emptyset$ and the subgradient is given by $\partial\phi(0) = \{-1, 1\}$

The spectral abscissa: The spectral abscissa is the function

$$\alpha : \mathbb{C}^{n\times n} \to \mathbb{R}$$

where $\alpha(M) = max(Re(\lambda(M)))$ with $\lambda(M) \in \mathbb{C}^n$ the set of eigenvalues of M. Burke and Overton [BO01] describe the regular subgradient of this function.

Teilaufgaben:

1. Implement a simple algorithm based on sampling the true gradient in a neighborhood of a matrix at which the spectral abscissa is not differentiable.
2. For such a matrix try to use the definition of subgradients and regular subgradients and the knowledge about those to numerically compute the subgradients of this matrix
3. Try to extract some theoretical information about the subgradient of the spectral abscissa

Literatur:

[BLO01] J. V. Burke, A.S. Lewis and M. L Overton, Approximating Subddifferentials by random sampling of gradients. *Math. Oper. Res.*, 27 (2002), 567?584

[BO01] J. V. Burke and M. L Overton, Variational Analysis of non-Lipschitz spectral functions. *Math. Program.*, 90:317–352, 2001.

[RW98] R. T. Rockafellar and R. J.-B Wets, Variational analysis. *Springer, Berlin* 1998

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