Max-Planck-Institut für
Dynamik komplexer technischer Systeme
Max Planck Institute for Dynamics of Complex Technical Systems

MAX-PLANCK-INSTITUT FÜR DYNAMIK KOMPLEXER TECHNISCHER SYSTEME MAGDEBURG

Thesis Topic

Research Group
"Computational Methods in Systems and Control Theory"

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Computational Methods in Systems and Control Theory

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Title

"Structure-Preserving Balancing of Matrix Pencils Arising in Linear-Quadratic Optimal Control"

Job Description

For the numerical solution of an eigenvalue problem

$$Ax = \lambda x$$

with $A \in \mathbb{R}^{n \times n}$ it is common practice to perform a preprocessing by balancing the matrix. Such a balancing is usually done in two steps:

(i) computation of a permutation matrix P such that

$$P^T AP = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ 0 & A_{22} & A_{23} \\ 0 & 0 & A_{33} \end{bmatrix},$$

where the matrices A_{11} and A_{33} are upper triangular so that their eigenvalues can be read off the diagonals;

(ii) computation of a diagonal matrix D such that the rows and the columns of the scaled matrix

$$D^{-1} A_{22} D$$

are as close in norm as possible.

This procedure is very useful to reduce the influence of roundoff errors during the eigenvalue computation, in particular if the magnitude differences of the entries of the matrix are very large.

Linear-quadratic optimal control problems often lead to structured generalized eigenvalue problems

$$\lambda \mathcal{S} x = \mathcal{H} x,$$

where the matrix pencil $\lambda \mathcal{S} - \mathcal{H}$ is either

(i) *even*, i.e., $\mathcal{S} = -\mathcal{S}^T$ and $\mathcal{H} = \mathcal{H}^T$; or

(ii) *skew-Hamiltonian/Hamiltonian*, i.e. $\mathcal{S}\mathcal{J} = -(\mathcal{S}\mathcal{J})^T$ and $\mathcal{H}\mathcal{J} = (\mathcal{H}\mathcal{J})^T$ with $\mathcal{J} = \left[\begin{smallmatrix} 0 & I_n \\ -I_n & 0 \end{smallmatrix}\right]$.

The task of this thesis project is to develop and implement a balancing strategy for *one* of the pencil structures above. The particular challenge is to find permutation and scaling matrices that preserve the even or skew-Hamiltonian/Hamiltonian structure of the pencils. This is crucial in order to be able to apply specialized eigenvalue algorithms to the balanced problem. This means that one has to restrict to special kinds of transformations, namely

(i) *congruence transformations* $\lambda\tilde{\mathcal{S}} - \tilde{\mathcal{H}} := \mathcal{U}^T(\lambda\mathcal{S} - \mathcal{H})\mathcal{U}$ in the even case; or

(ii) $\mathcal{J}$*-congruence transformations* $\lambda\tilde{\mathcal{S}} - \tilde{\mathcal{H}} := \mathcal{J}^T\mathcal{U}^T\mathcal{J}(\lambda\mathcal{S} - \mathcal{H})\mathcal{U}$ in the skew-Hamiltonian/Hamiltonian case.

Finally, a FORTRAN 77 subroutine in the style of SLICOT[1] codes should be developed to compare the accuracy of the results for the balanced and unbalanced problem.

References

[1] P. Benner. *Symplectic balancing of Hamiltonian matrices*, SIAM J. Sci. Comput., 22(5):1885–1905, 2001.

[2] P. Benner, V. Sima, and M. Voigt. *FORTRAN 77 subroutines for the solution of skew-Hamiltonian/Hamiltonian eigenproblems, Part I – Algorithms and applications*, Preprint MPIMD/13-11, Max Planck Institute Magdeburg, 2013.

[3] P. Benner, V. Sima, and M. Voigt. *FORTRAN 77 subroutines for the solution of skew-Hamiltonian/Hamiltonian eigenproblems, Part II – Implementation and numerical results*, Preprint MPIMD/13-12, Max Planck Institute Magdeburg, 2013.

[4] B. N. Parlett and C. Reinsch. *Balancing a matrix for calculation of eigenvalues and eigenvectors*, Numer. Math., 13(4):293–304, 1969.

[5] C. Schröder. *Palindromic and Even Eigenvalue Problems – Analysis and Numerical Methods*, Dissertation, Institut für Mathematik, Technische Universität Berlin, 2008.

[6] R. C. Ward. *Balancing the generalized eigenvalue problem*, SIAM J. Sci. Stat. Comput., 2(2):141–152, 1981.

Job Requirements
Recommended: Numerical Linear Algebra (Eigenvalue Problems).
Desirable: Systems and Control Theory, Scientific Computing.

Degree
Diplom or Master

[1] `http://www.slicot.org`